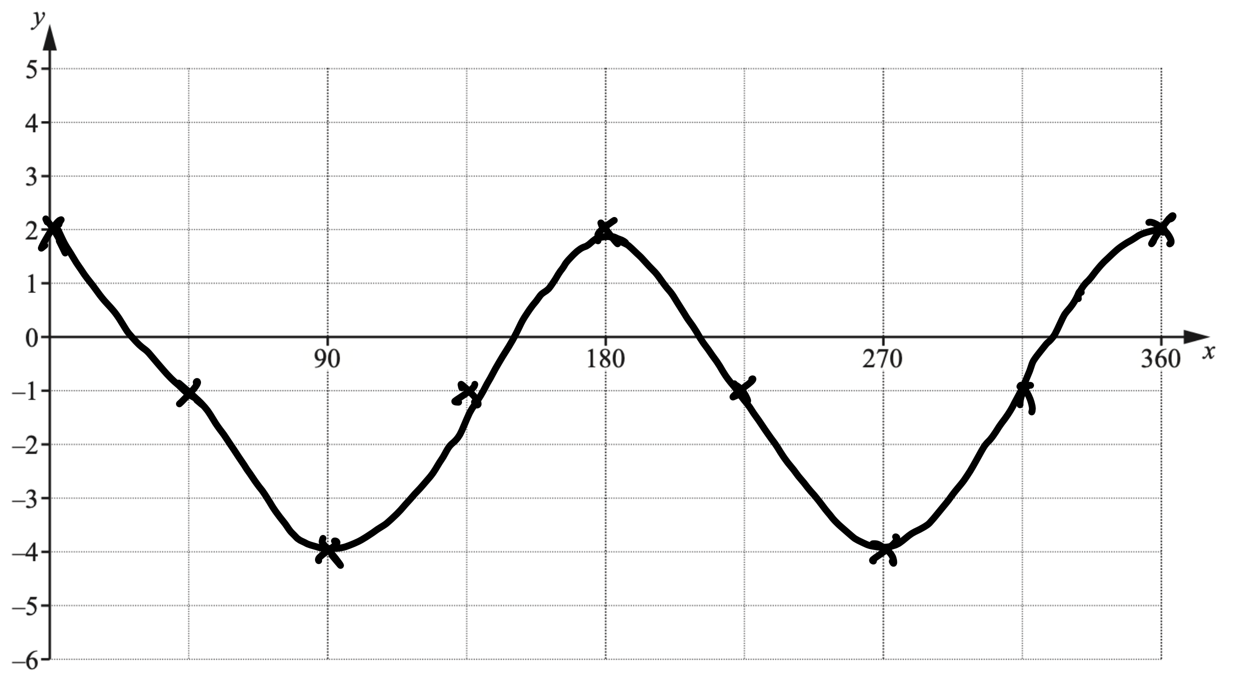
A Maths CH 1 to 9

_____/74 marks

1. (a) On the axes below, sketch the graph of $y = 3cos2x - 1$, for $0° \leq x° \leq 360°$.



[3]

(b) Given that $y = 4sin\ 6x$, write down

(i) the amplitude of *y*,

4

[1]

(ii) the period of *y*.

[1]

60°

2. (i) Write $x^2 - 9x + 8$ in the form $(x - p)^2 - q$.where *p* and *q* are constants.

$(x - 4.5)^2 - (4.5)^2 + 8$ [2]

$= (x - 4.5)^2 - 12.25$

$p = 4.5,\ q = 12.25$

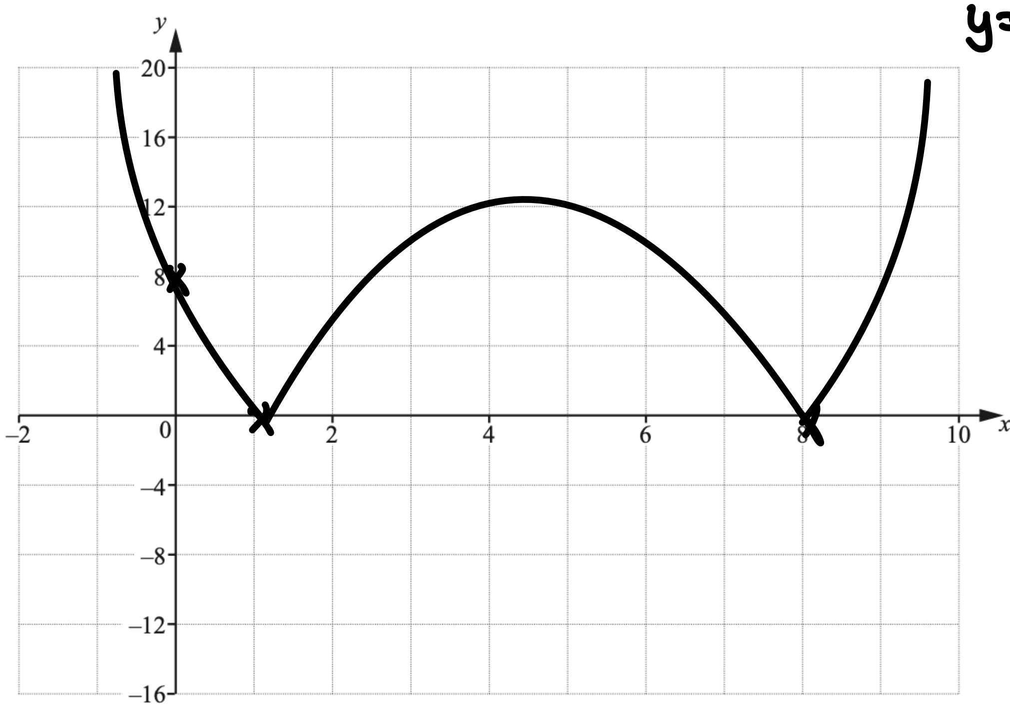
(ii) Hence write down the coordinates of the minimum point on the curve $y = x^2 - 9x + 8$.

$(4.5, -12.25)$ [1]

(iii) On the axes below, sketch the graph of $y = |x^2 - 9x + 8|$, showing the coordinates of the points where the curve meets the coordinate axes.

$y = 0, x = 8$ or 1

$x = 0, y = 8$



[3]

(iv) Write down the value of *k* for which $k = |x^2 - 9x + 8|$ has exactly 3 solutions.

$k = 12.25$

[1]

3. (a) $f(x) = 3 - cos2x$ for $0 \le x \le \frac{\pi}{2}$.

i. Write down the range of f.

[2]

$3 - \cos(0) = 3 - 1 = 2$

$3 - \cos(\pi) = 3 - -1 = 4$

$2 \le f \le 4$

ii. Find the exact value of $f^{-1}(2.5)$.

[3]

$y = 3 - \cos 2x$

$x = 3 - \cos 2y$

$\cos 2y = 3 - x$

$2y = \cos^{-1}(3 - x)$

$y = \dfrac{\cos^{-1}(3 - x)}{2}$

$f^{-1}(2.5) = \dfrac{\cos^{-1}(3 - 2.5)}{2} = \dfrac{\cos^{-1}(0.5)}{2}$

$= \frac{1}{6}\pi$

(b) $g(x) = 3 - x^2$ for $x \in R$.

Find the exact solutions of $g^2(x) = -6$

[4]

$g^2(x) = 3 - (3 - x^2)^2$

$-6 = 3 - (3 - x^2)^2$

$(3 - x^2)^2 = 9$

$3 - x^2 = 3$ or -3

$3 - x^2 = 3$

$0 = x^2$

$x = 0$

$3 - x^2 = -3$

$6 = x^2$

$x = \pm\sqrt{6}$

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4. Solve the equation $|5x - 3| = -3x + 13$.

[3]

$5x - 3 = -3x + 13$ or $5x - 3 = 3x - 13$

$8x = 16$

$x = 2$

$5x = 3x - 10$

$2x = -10$

$x = -5$

5. Solve the simultaneous equations

$$log_2(x + 2y) = 3,$$

$$log_2 3x - log_2 y = 1.$$

[5]

$\log_2 (x + 2y) = 3$

$x + 2y = 8$

$x = 8 - 2y$

$\log_2 {}^{3x}/_{y} = 1$

$3x/y = 2$

$3x = 2y$

$3(8 - 2y) = 2y$

$24 - 6y = 2y$

$24 = 8y$

$y = 3$

$x = 8 - 2(3)$

$= 8 - 6$

$= 2$

6. Variables x and y are such that when y^2 is plotted against e^{2x} a straight line is obtained which passes through the points (1.5, 5.5) and (3.7, 12.1). Find

   a. y in terms of e^{2x}, [3]

$$y^2 = me^{2x} + c$$

$$m = \frac{12.1 - 5.5}{3.7 - 1.5} = 3$$

$$y^2 = 3e^{2x} + c$$

$$5.5 = 3(1.5) + c$$

$$c = 1$$

$$y^2 = 3e^{2x} + 1$$

$$y = \sqrt{3e^{2x} + 1}$$

   b. the value of y when $x = 3$, [1]

$$y = \sqrt{3e^{2(3)} + 1}$$

$$y = \sqrt{3e^{6} + 1}$$

$$y = 34.8$$

   c. the value of x when $y = 50$. [3]

$$50 = \sqrt{3e^{2x} + 1}$$

$$2500 = 3e^{2x} + 1$$

$$2499 = 3e^{2x}$$

$$833 = e^{2x}$$

$$\ln 833 = 2x$$

$$x = \frac{\ln 833}{2}$$

7. (a) Solve $2sin\,(x + \frac{\pi}{4}) = \sqrt{3}$ for $0 < x < \pi$ radians. [3]

$$2\sin\left(x+\frac{\pi}{4}\right)=\sqrt{3} \qquad \pi/4 < x+\pi/4 < 5\pi/4$$

$$\sin\left(x+\frac{\pi}{4}\right)=\sqrt{3}/2$$

$$x+\pi/4 = \sin^{-1}(\sqrt{3}/2),\ \pi - \sin^{-1}(\sqrt{3}/2)$$

$$= 1/3\,\pi,\ 2/3\,\pi$$

$$x = 1/12\,\pi,\ 5/12\,\pi$$

(b) Solve $3sec\,y = 4\,cosec\,y$ for $0° < y < 360°$. [3]

$$\frac{3}{\cos y} = \frac{4}{\sin y}$$

$$\frac{3\sin y}{\cos y} = 4$$

$$3\tan y = 4$$

$$\tan y = 4/3$$

$$y = \tan^{-1}(4/3),\ 180 + \tan^{-1}(4/3)$$

$$= 53.1°,\ 233°$$

(c) Solve $7\,cot\,z - tan\,z = 2cosec\,z$ for $0° < z < 360°$.

[6]

$$7\left(\frac{\cos x}{\sin x}\right) - \frac{\sin x}{\cos x} = \frac{2}{\sin x}$$

$$\frac{7\cos x - 2}{\sin x} = \frac{\sin x}{\cos x}$$

$$7\cos^2 x - 2\cos x = \sin^2 x$$

$$7\cos^2 x - 2\cos x = 1 - \cos^2 x$$

$$8\cos^2 x - 2\cos x - 1 = 0$$

$$(2\cos x - 1)(4\cos x + 1) = 0$$

$$\cos z = 1/2 \text{ or } -1/4$$

$\cos z = 1/2$

$z = \cos^{-1}(1/2),\ 360 - \cos^{-1}(1/2)$

$z = 60°,\ 300°$

$\cos z = -1/4$

$z = 180 - \cos^{-1}(1/4),\ 180 + \cos^{-1}(1/4)$

$= 104.48°,\ 255.5°$

8. The line $y = 2x + 5$ intersects the curve $y + xy = 5$ at the points *A* and *B*. Find the coordinates of the point where the perpendicular bisector of the line *AB* intersects the line *y* = *x*.

[9]

$y = 2x + 5$

$y + xy = 5$

$y(1+x) = 5$

$y = \frac{5}{1+x}$

$2x + 5 = \frac{5}{1+x}$

$(2x+5)(1+x) = 5$

$2x + 2x^2 + 5 + 5x = 5$

$2x^2 + 7x = 0$

$x(2x+7) = 0$

$x = 0$ or $x = -7/2$

$\therefore y = 5$ or -2

$m_{AB} = \frac{5 - -2}{0 - -7/2} = 2$

$-1/2$

midpoint AB: $\left(\frac{0 - 7/2}{2}, \frac{5 - 2}{2}\right)$

$= \left(-7/4, 3/2\right)$

$y - 3/2 = -1/2\,(x + 7/4)$

$y = -1/2\,x + 5/8$

$x = -1/2\,x + 5/8$

$3/2\,x = 5/8$

$x = 5/12$

$\rightarrow (5/12, 5/12)$

9. The remainder obtained when the polynomial $p(x) = x^3 + ax^2 - 3x + b$ is divided by $x + 3$ is twice the remainder obtained when p(*x*) is divided by $x - 2$. Given also that p(*x*) is divisible by $x + 1$, find the value of *a* and of *b*.

[5]

$p(-3) = 2R$ 1 $\quad 2R = -27 + 9a + 9 + b$

$p(2) = R$ 1 $\quad 2R = 9a + b - 18$

$p(-1) = 0$ 1

$R = 8 + 4a - 6 + b$

$\times 2 \quad R = 4a + b + 2$

$2R = 8a + 2b + 4$

$8a + 2b + 4 = 9a + b - 18$

$8a + 2b + 22 = 9a + b$

$22 = a - b$

$22 + b = a$

$0 = -1 + a + 3 + b$

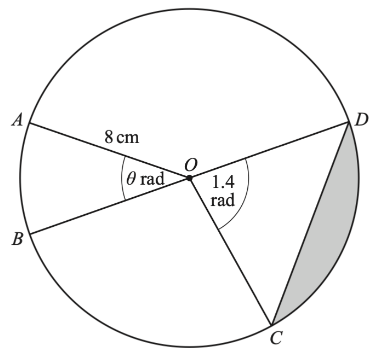
$0 = -1 + 22 + b + 3 + b$

$0 = 24 + 2b$

$-2b = 24$

$b = -12 \quad \therefore a = 10$

10.



The diagram shows a circle with centre *O* and radius 8 cm. The points *A*, *B*, *C* and *D* lie on the circumference of the circle. Angle *AOB* = θ radians and angle *COD* = 1.4 radians. The area of sector *AOB* is 20 cm^2

a. Find angle θ.

$20 = \frac{1}{2} r^2 \theta$

$40 = 8^2 \theta$ $\longrightarrow$ $\theta = 5/8$

[2]

b. Find the length of the arc *AB*.

Arc $= r\theta$

$= 8(5/8)$

$= 5\text{cm}$

[2]

c. Find the area of the shaded segment.

Sector ODC $= \frac{1}{2} r^2 \theta$

[3]

$= \frac{1}{2}(8)^2(1.4)$

$= 224/5$

$\triangle ODC = \frac{1}{2} ab \sin C$

$= \frac{1}{2}(8)(8)\sin(1.4) = 31.53$

Shaded: $\frac{224}{5} - 31.53$

$= 13.3\text{cm}^2$

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12. Determine the set of values of *k* for which the equation $(3 - 2k)x^2 + (2k - 3)x + 1 = 0$ has no real roots.

[5]

$b^2 - 4ac < 0$

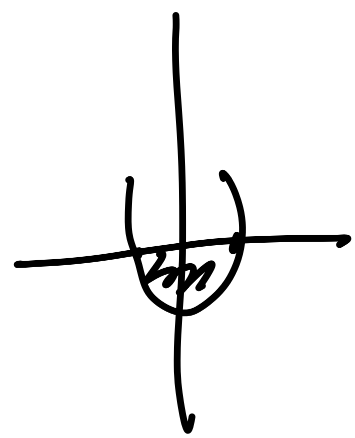
$(2k-3)^2 - 4(3-2k)(1) < 0$

$4k^2 - 12k + 9 - 12 + 8k < 0$

$4k^2 - 4k - 3 < 0$

$(2k-3)(2k+1) < 0$

$k = \frac{3}{2}$ or $-\frac{1}{2}$



$-\frac{1}{2} < k < \frac{3}{2}$